# Are You More Popular Than Your Friends?

## On average, that is

For clarity:

- Define popularity as number of friends
- Consider a person $i$
- Let $m_i$ be the number of friends of $i$ who are more popular than $i$
- Let $l_i$ be the number of friends of $i$ who are less popular than $i$
- For most $i$, $m_i > l_i$

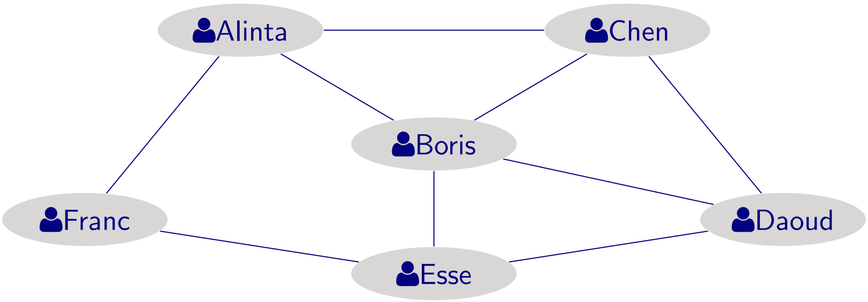
For most $i$, is $m_i > l_i$, $m_i = l_i$, or $m_i < l_i$?
Number of people $i$ whose $m_i > l_i$ versus number of people $i$ whose $m_i < l_i$

# No!

## Hint

- You may be, but most people are not
- Model a person as a vertex in a graph: $N$ vertices
- Model friendship as an undirected edge
- The average degree of the graph indicates the average friendliness
- Consider two people, one above and one below the average
- Thus their degrees relate $d_{\text{popular}} > d_{\text{unpopular}}$
- Informally, $d_{\text{popular}}$ people suffer by having a popular friend and $d_{\text{unpopular}}$ people gain by having a less popular friend
- But $d_{\text{popular}} > d_{\text{unpopular}}$

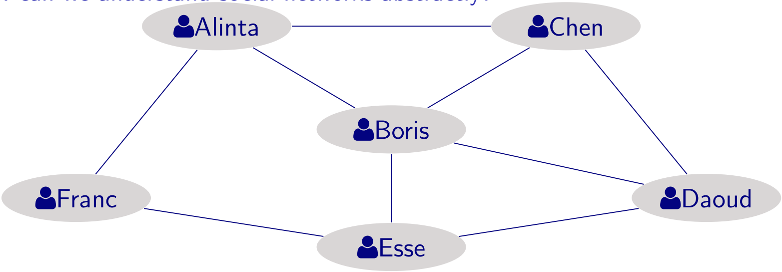
# Counting Friends




| | Friends | Friends' friends average | |
|---|---|---|---|
| Alinta | 3 | 3 | |
| Boris | 4 | 3 | more popular than friends |
| Chen | 3 | 3 | |
| Daoud | 3 | $3\frac{1}{3}$ | less popular than friends |
| Esse | 3 | 3 | |
| Franc | 2 | 3 | less popular than friends |

# Sociometrics and Structure

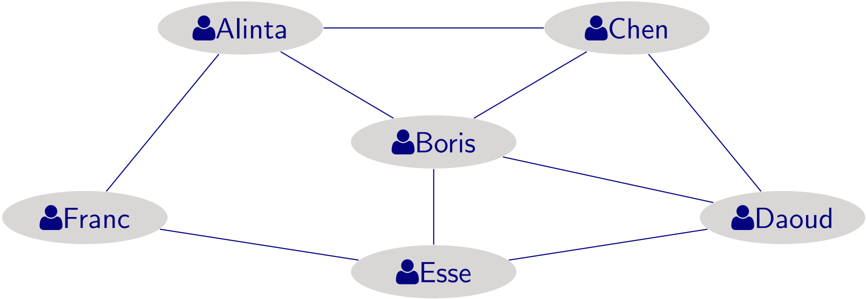
## How can we understand social networks abstractly?




- Set of edges: no good for analysis because it doesn't say much
- Entire graph: too complex and varied
- Triad: Consider nodes three at a time
  - Typically, unlabeled edges (or all have same label)
  - Typically, undirected edges but not always
  - A small number of possible triads
  - See how they are distributed over a network
- More complex subgraphs: increasingly studied

# Exercise: Cliquishness

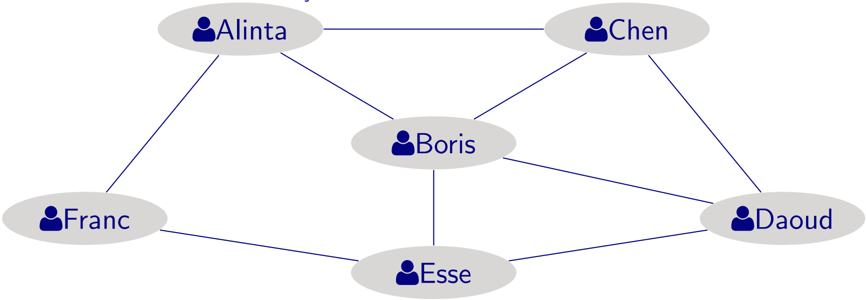
Clique (pronounced click): $\sim$ closed social group




- How would you define it?
  - As a property of an individual
  - As a property of a network
- How would you define it mathematically?

# Cliquishness

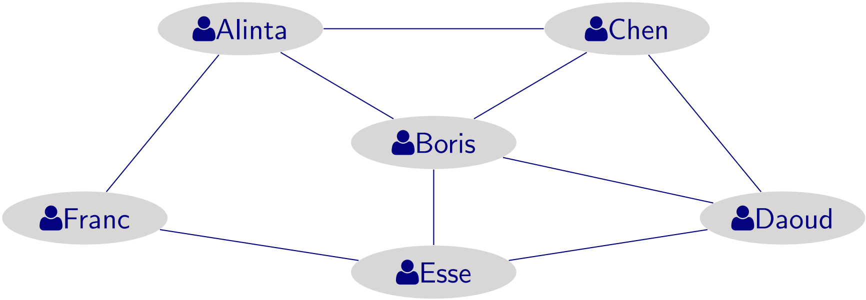
## Typical definition in network analysis




- Clique: complete subgraph
- Cliquishness: How many of the possible triads are closed
- Evaluate for each vertex
  - How many of its friends are mutual friends
  - Ratio of actual mutual friendships to possible mutual friendships
- For a network, average over its vertices

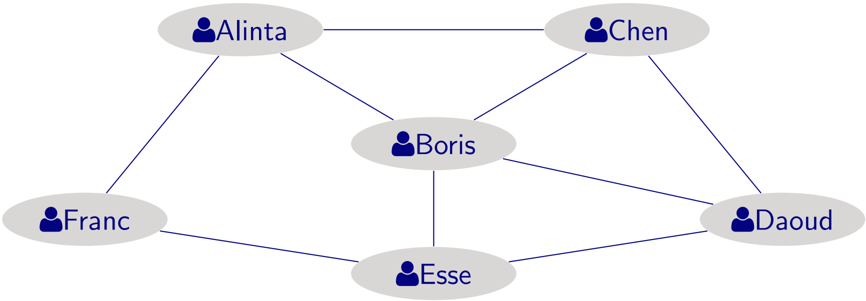
# Exercise: Estimate Importance or Influence of a Vertex

Also called centrality measures



# Centrality Measures

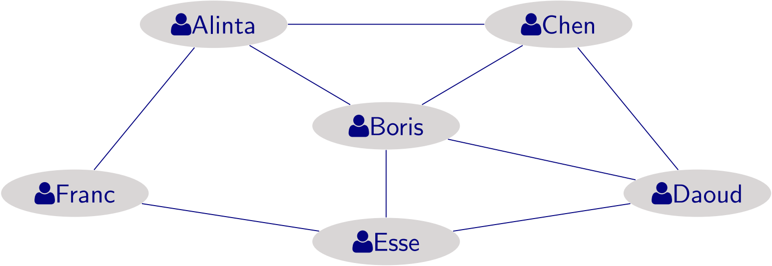
## Almost an unlimited supply of metrics




- Degree
- Removal would increase path length for others the most
- Betweenness centrality: How many shortest paths it lies on
- Has minimum of maximum path length to others

# Weak Ties

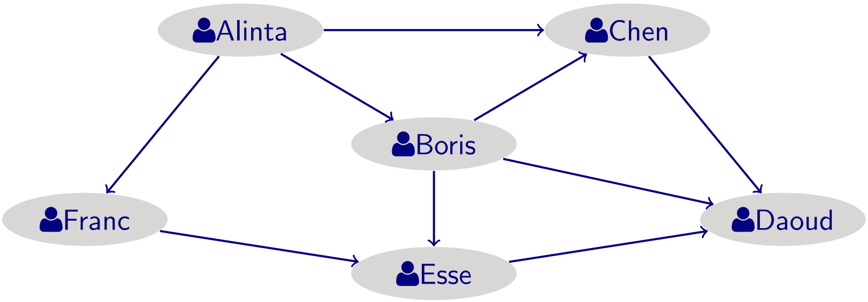
## Mark Granovetter




- Distinguish ties (undirected edges) based on strength
- Weak
  - Infrequent interactions
  - Low intimacy
  - Someone you meet occasionally, e.g., acquaintance
- Weak ties often connect otherwise disjoint parts of the network
- More likely to lead to surprising knowledge
- Effective in producing job referrals

# Consider Directed Networks

Give some natural examples




- Define influence for two of your examples

# Simplified Page Rank

Influential people are those whom influential people link to (e.g., follow on Twitter)

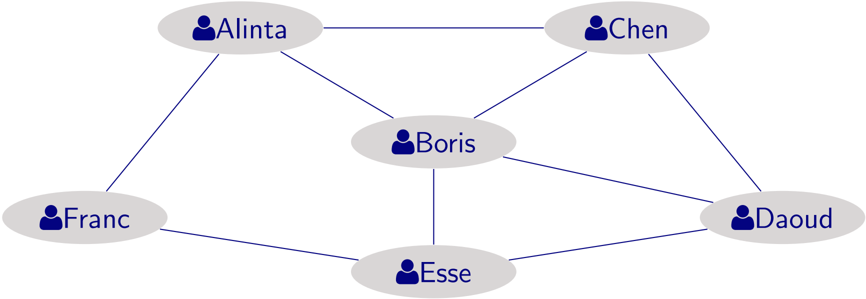
- Model Twitter as a graph $(V, E)$, $|V| = N$ authors
  - Ignoring the tweets
- A random search lands you at an author with probability $e$
  - Assuming authors are uniformly likely to be found
- Each author Alice gets a vote
  - Equal to Alice's rank
  - Distributed equally among all the authors she follows

$$R(x) = \frac{e}{N} + (1-e) \sum_{y:(y,x) \in E} \frac{R(y)}{out(y)}$$

- Naïve method to compute
  - Initialize $R(x)$ to $1/N$
  - Iterate until fixed point

# Social Network Dynamics

## The previous metrics were static: for a fixed graph




- How would people act to create friendships or follow others?
- Describe a story by which the people above could end up with the above network
- What kinds of networks are likely to emerge?

# Emergent Properties

Depend on processes of attachment
Knowledge and incentives of those deciding
Easiest to think of for directed graphs because they involve one party's decision making

- Preferential attachment
  - A new entrant will connect with others proportional to their current degree
  - Rich get richer
  - Purely an abstract mathematical model
- For humans, many factors come up
  - Alignment of interests
  - Coolness factor?
  - Requesting, giving, following referrals

# Random Graphs

Exercise: Define one

Give a method for producing one

# Random Graphs

## Erdös-Rényi

- Anonymous vertices
- Unlabeled, undirected edges
- Consider $n$ vertices and $e$ edges
  - $\binom{n}{2} = \frac{n(n-1)}{2}$ edges are possible
  - Uniform-randomly choose $e$ out $\binom{n}{2}$

# Exercise: Erdös-Rényi Random Graphs

Use $n$ vertices and $e$ edges

- What would be the average degree?
- How might the degree be distributed (in intuitive terms) across the $n$ vertices?
- Relative to graphs of $n$ vertices and $e$ edges and in qualitative terms (low, medium, high)
    - What would be its cliquishness (called *clustering* by Watts & Strogatz)?
    - What would be its average path length (all-pairs shortest distance)?

# Exercise: Regular Graphs

Like a ring (of $n$ vertices and $e$ edges)
A familiar ring has each vertex of degree two
Imagine rings with larger degrees

- What would be the average degree?
- How might the degree be distributed (in intuitive terms) across the $n$ vertices?
- Relative to graphs of $n$ vertices and $e$ edges and in qualitative terms (low, medium, high)
  - What would be its cliquishness (called clustering in this paper)?
  - What would be its average path length (all-pairs shortest distance)?

# Small-World Networks

## Watts and Strogatz

- Real-life networks (social, economic, physical, biological) exhibit
  - High clustering (cliquishness)
  - Low average path length
- Not like Erdös-Rényi random graphs
  - Indicate some bias in connectivity
- Not like regular graphs

# Constructing a Small-World Network

## Watts and Strogatz

- Assume $e = 2n$ or $4n$ or ...
- Begin from a ring
- With probability $p$ select an edge and holding one vertex fixed, reconnect the other
  - $p = 0$: No change, so regular
  - $p = 1$: Maximum change, so random
  - In the middle: Interesting cases
- Characteristic path length, as a function of $p$: $L(p)$
  - Global property
- Clustering, as a function of $p$: $C(p)$
  - Local property

# Exercise: Small-World Networks

Suppose a regular graph is rewired with low $p \sim 0.1$

- Consider a regular graph with $n = 1000$ and $e = 2000$
- How would its clustering change?
- How would its average path length change?

# Exercise: Small-World Networks

Plot $L(p)$ and $C(p)$ as functions of $p$

In qualitative terms, what's the shape of these curve

Normalize $L$ as $\frac{L(p)}{L(0)}$ and $C$ as $\frac{C(p)}{C(0)}$ to relativize to regular graphs

# Wayfinding

## How would you find a way in geographical space

- Back to your hotel in a city you are visiting?
- Via Uffici del Vicario, 40
- Giolitti
- That gelato place in Rome where Audrey Hepburn and Barack Obama stopped
- How would you search for the Titanic sunk under the Atlantic?

# Wayfinding

- How would you find a way in geographical space?
  - Look up on a map
  - Head in a promising direction
  - Apply geographical knowledge: follow a river downstream or upstream
  - Ask someone
- Searching for the Titanic
  - Brute force search?
  - Actual approach
    - Locate debris trail running North-South
    - Move North along it

# Wayfinding in a Social Network

## Social geography

### John Donne: No man is an island

- What are some example uses of social wayfinding
- Compare with geographical space
- How might Facebook do it?
- How would you do it as an individual user?

# Exercise: Propose an Algorithm for Decentralized Wayfinding

# Algorithms for Decentralized Wayfinding

- Flooding
- Referrals
  - Who to ask
  - Whom to respond to
  - What response to give
  - How to assess responses
  - How to learn from each episode

# Dynamism in Referral Networks: Evolution

Depends on how the participants explore and learn

# Social Mobilization

## Getting people to act

Exercise: come up with a task and a possible solution

- Clean up a park or a beach
- Donate blood
- Help locate a suspect in a crime or terror attack
- Help locate an cognitively challenged person who wandered off from home

# Programming Competitions

## What motivates you to participate?

# Balloon Challenge

DARPA: Defense Advanced Research Projects Agency
Exercise: how would you do it?

- Release 10 balloons in the continental US
- Competition between teams
- Whichever team locates (visually) all 10 first gets $50k
- Two groups being motivated
  - By DARPA: Researchers who would pursue the competition
  - By researchers: Members of the public to help them

# Balloon Challenge Outcomes

- Winning team
  - Employed referrals
  - Reward for first person to find a balloon
  - Exponentially decaying rewards for chain of referrers
- Other approaches
  - Donation to charity
  - Use of social media personality to tweet about it

# Exercise: Identify Limitations of these Approaches

Give specific examples