Rationality

- Basis for understanding interactions among autonomous parties
- Many questions reduce to resource allocation
- What is an optimal or correct resource allocation

Mechanism Design

- *Mechanism:* a set of rules of an environment under which agents operate
  - Honor systems
  - Honor systems with social censure (as a penalty)
  - Auctions
  - Paying taxes (voluntary, but with selective audits and severe penalties for violators)
- How do the above compare?
- *Mechanism design:* Creating a mechanism to obtain desired system-level properties, e.g., participating agents interact productively and fairly

Example Mechanism: Puzzle from the Talmud

Given two horses to be raced for a mile

- Owner of horse proved *faster* wins a reward
  - Each owner is or hires a jockey
  - The horses are raced against each other
  - The winner of the race wins
- Owner of horse proved *slower* wins a reward
  - Might consider rewarding the loser of a race, but such a race won't terminate because each rider will want to go slower than the other

Economic Abstractions

- Support achieving optimal resource allocations
- Capture a notion of autonomy and rationality
- Provide a basis for achieving some contractual behaviors, especially in helping
  - An individual agent decide what to do
  - Agents negotiate
- Incomplete by themselves

How Can Trade Work?

Whether barter or using money

- Why would rational agents voluntarily participate?
- Both cannot possibly gain; or can they?
- Consider the following. Would you trade
  - A dollar bill for another dollar bill?
  - A US dollar for x Euros?
  - Money for a bottle of drinking water?
  - A bottle of drinking water for money?

It comes down to your valuations: differences in valuations make trade possible

Kinds of Valuations

How do agents place values of goods?

- *Independent (and private):* Agents value goods in a manner that is unaffected by others
  - Consume or use: cake
- *Common:* Agents value goods entirely based on others' valuations, leading to symmetric valuations
  - Resale: treasury bills
- *Correlated:* Combination of above
  - Automobile or house

Markets Introduced

Compare stock with specific real-estate

- Can be
  - *Public*
  - *Private:* part of restricted exchanges
- Can restrict kinds of goods traded
  - *Endogenous:* NASDAQ
  - *Exogenous:* eBay, where physical goods are traded outside the scope of the market
- Offer some form of nonrepudiation

Centrality of Prices

A price is a scalar: easy to compare

- The computational state of a market is described completely by current prices for the various goods
- Communications are between each participant and the market, and only in terms of prices
- Participants reason about others and choose strategies entirely in terms of prices being bid

Functions of a Market

- Provides this information to participants
- Takes requests (buy, sell bids) from participants, enforcing rules such as bid increments and time limits
- Decides outcome based on messages from participants, considering rules such as reserve prices, ...

Achieving Equilibrium

When supply equals demand

- At equilibrium, the market has computed the allocation of resources
  - Dictates the activities and consumptions of the agents
- Under certain conditions, a simultaneous equilibrium of supply and demand across all goods exists
  - That is, the market "clears"
  - Reachable via distributed bidding
  - *Pareto optimal:* you cannot make the allocation better for one agent without making it worse for another

Pareto Optimality

- Allocation: how resources are allocated to different parties
- Think of a vector of allocations, one dimension for each participant
- An allocation is Pareto optimal if improvements along any dimension must be accompanied by a reduction along another dimension

Auctions in Markets

Computational mechanism to manage supply and demand by computing a price to trade at

- Exchange common object (money) for goods
  - Ascending (English) vs. Descending (Dutch)
  - Silent (auctioneer names a price; bids are silent) vs. outcry (bids name prices; auctioneer listens)
  - Hidden identity or not
  - Combinatorial: involve *bundles* or sets of goods

English Auction

Buyers bid for an item

- Prices start low and increase
- Highest bidder gets the object and pays the price bid
- Variations:
  - Minimum bid increment
  - Reserve price (no sale if too low)
  - Limited time

Dutch Auction

- Price “clock” or counter starts high and winds down
- First to stop the clock wins and pays the price on the clock
- In other words, the highest bidder wins and pays the price bid

Fish Market Auction

Imagined scenario is based on a Spanish fish market

- Auctioneer calls out prices
  - If two or more bidders
    - repeat with higher price
  - If no bidders
    - repeat with lower price

Winner's Curse: 1

- If you just won an English auction
- You just paid \$x for something
- How much can you sell it for?
- Obviously, you will be able to sell it for ...

Not quite a curse if inherently valuable to you, but perhaps could have obtained the item for less

Winner's Curse: 2

Sealed bid; no resale

- A group of mutually independent people estimate the values of different goods and bid accordingly
- Assume that the group is smart
  - The average is about right as an estimate of the true value
- The winner bid the maximum

Suckers' Auction

Consider two bidders bidding for $1 currency

- Bid in increments of 10¢
- Highest bidder wins
- Both bidders pay (i.e., loser also pays)
- Once you are in, can you get out?
  - The myopically rational strategy is to bid
  - The outcome is not pleasant

Sealed Bid First-Price Auction

Also known as *tenders*: bidding to buy

- One-shot bidding without knowing what other bids are being placed
- Used by governments and large companies to give out certain large contracts (lowest price quote for stated task or procurement)
  - All bids are gathered
  - Auctioneer decides outcomes based on given rules (e.g., highest bidder wins and pays the price it bid)

Vickrey Auction

- Second-price sealed bid auction
- Highest bidder wins, but pays the *second* highest price

Pricing

Intuition: Allocate resources to those who value them the most

- Fixed: slowly changing, based on various criteria
  - *Flexibility:* (restrict rerouting or refundability in air travel)
  - *Urgency:* (convenience store vs. warehouse)
  - *Customer preferences* (coupons: price-sensitive customers like them; others pay full price)
  - *Demographics*
  - *Artificial* (Paris Metro, Delhi "Deluxe" buses)
  - *Predicted demand* (New York subway, phone rates)
- Dynamic: rapidly changing, based on actual demand and supply

M^{th} and $(M+1)^{st}$ Price Auctions: 1

- L = M+N single-unit sealed bids, not continuously cleared
  - M sell bids
  - N buy bids
- M^{th} price clearing rule
  - Price = M^{th} highest among all L bids
  - English: first price; M=1
    - Seller's reserve price is the sole sell bid (assume minimum value, if no explicit reserve price)
- $(M+1)^{st}$ price clearing rule
  - Price = $(M+1)^{st}$ highest among all L bids
  - Vickrey: second price; M=1

M^{th} and $(M+1)^{st}$ Price Auctions: 2

The M^{th} and $(M+1)^{st}$ prices delimit the equilibrium price range, where supply and demand are balanced

- Above M^{th} price: no demand from some buyers
- Below $(M+1)^{st}$ price: no supply from some sellers

Concepts About Matching

Buy and sell bids can be matched in various ways, which support different properties

- *Equilibrium prices:* those at which supply equals demand, also known as *market price*
- *Individually rational:* each agent is no worse off participating than otherwise
- *Efficient:* No further gains possible from trade (agents who value goods most get them): i.e., Pareto optimal
- *Uniform price:* Multiple units, if simultaneously matched, are traded at the same price
- *Discriminatory:* Trading price for each pair of bidders can be different
- *Incentive compatible:* Agents optimize their expected utility by bidding their true valuations

Incentive Compatibility

Incentives are such it is rational to tell the truth

- Ramification: Agents can ignore subtle strategies and others' decisions: hence simpler demands for knowing others' preferences and reasoning about them
- Basic approach: payoff depends not on decisions (bids) by self
- Example: Vickrey (second-price sealed bid) auctions for independent private valuations
  - Underbid: likelier to lose, but price paid on winning is unaffected by bid
  - Overbid: likelier to win, but may pay more

Economic Rationality

- Space of alternatives or outcomes
- Each agent has some ordinal (i.e., sorted) preferences over the alternatives, captured by a binary relation, $\succ$
  - $\succ$ is a strict ordering
    - Asymmetric, Transitive (implies irreflexive)
  - $\succ$ is not total
  - Another binary relation, $\sim$, captures indifference

Lotteries

Probability distributions over outcomes or alternatives (add up to 1)

- In essence, define potential outcomes
  - Flip a coin for a dollar: [0.5: \$1; 0.5:–\$1]
  - Buy a \$10 ticket to win a car in a raffle: [0.0001: car$-$\$10; 0.9999: $-$\$10]
  - Four choices: $[p : A; q : B; r : C; 1 - p - q - r : D]$

Using Lotteries

Infer (rational) agents' preferences based on their behavior with respect to the lotteries

- What odds will a specific person accept?
- For example, [0.01: car−$10; 0.99: −$10]

Properties of Lotteries

- Substitutability of indifferent outcomes
  - If $A \sim B$, then $[p : A; (1 - p) : C] \sim [p : B; (1 - p) : C]$
- Monotonicity (for preferred outcomes)
  - If $A \succ B$ and $p > q$, then $[p : A; (1 - p) : B] \succ [q : A; (1 - q) : B]$
- Decomposibility (flatten out a lottery)
  - Compound lotteries reduce to simpler ones
  - $[p : [q : A; 1 - q : B]; 1 - p : C] = [pq : A; p - pq : B; 1 - p : C]$

Expected Payoff

- Expresses the value of a lottery as a scalar (i.e., in monetary terms)
- Expected payoff is sum of utilities weighted by probability
- Utilities are *not* proportional to monetary amounts, but assume so for this example
  - Calculate for [0.0001: car−\$10; 0.9999: −\$10] where the car is worth \$25,010

Completeness of Preferences

Same as indifference being an equivalence relation

- Given outcomes A and B
  - $\preceq$ means nonstrict preference
  - Either $A \preceq B$ or $B \preceq A$
- That is, $A \sim B$ if and only if $A \preceq B$ and $B \preceq A$
- Thus, $\sim$ is an *equivalence* relation
  - *Reflexivity:* $A \sim A$
  - *Symmetry:* $A \sim B$ implies $B \sim A$
  - *Transitivity:* $(A \sim B$ and $B \sim C)$ implies $A \sim C$

Continuity of Preferences

- $A \succ B \succ C$ implies that there is a probability p, such that
  - $[p : A; 1 - p : C] \sim B$
  - Consider A, B, and C to be ice-cream, yogurt, and cookies, respectively
- Informally, this means we can price alternatives in terms of each other
- Is this reasonable in real life? Why or why not?

Utility Functions

One per agent

- Map each alternative (outcome) to a scalar (real number)
  - U: $\{\text{alternatives}\} \rightarrow \mathscr{R}$
- For agents with irreflexive, transitive, complete, continuous preferences, there is a utility function U such that
  - $U(A) > U(B)$ implies $A \succ B$
  - $U(A) = U(B)$ implies $A \sim B$
  - $U([p : A; 1-p : C]) = p \times U(A) + (1-p) \times U(C)$ (weighted sum of utilities)

Risk: 1

- According to the above, two lotteries with the same expected payoff would have equal utility
- In practice, risk makes a big difference
  - Raffles
  - Insurance
  - Business actions with unpredictable outcomes

Risk: 2

The utility of an outcome depends not only on the outcome but also on the distribution of outcomes

- Consider two lotteries
  - $L_1 = [1 : x]$
  - $L_2 = [p : y; 1 - p : z]$
  - Where $x = py + (1 - p)z$. That is, L_1 and L_2 have the same expected payoff
- An agent's preferences reflect its attitude to risk
  - *Neutral:* $U(L_1) = U(L_2)$
  - *Averse:* $U(L_1) > U(L_2)$
  - *Seeking:* $U(L_1) < U(L_2)$

Beyond Simple Utility

Other factors besides expected payoff and risk are relevant in real life

- Total deal value: $10 discount for a t-shirt vs. for a car
  - Compare with Tversky and Kahneman's studies
- Current wealth: 1^{st} million vs. 10^{th} million
- Altruism or lack thereof

Simplifying Assumptions

- Participants are risk neutral
  - Willing to trade money for any of their resources at a price independent of how much money they already have
- Participants know their valuations, which are independent and private

Sharing Resources

Leads to social choice theory

Consider two scenarios for sharing—only requirement is that the parties agree on the split

- *Splitting a dollar:* relative sizes are obvious. Should splits consider the relative wealth of the splitters? Should splits consider the tax rates of the splitters?
- *Sharing a cake:* relative sizes and other attributes (e.g., amount of icing) can vary—several cake-cutting algorithms exist

Pareto Optimality

A distribution of resources where no agent can be made better off without making another agent worse off

- Example: A has goods g and values g at \$1; B values g at \$3
  - It is Pareto optimal for B to buy g at a price between \$1 and \$3, say \$2.50
  - A's gain: \$2.50–\$1 = \$1.50
  - B's gain: \$3–\$2.50 = \$0.50
- No further gains can be made from trade

Computing Pareto Optimal Allocations

- Setting
  - Private valuations
  - No central control
- Design mechanisms that are efficient and where participants have an incentive to bid their private values
  - Buyers and sellers are symmetrical: may need to flip a coin

Vickrey Incentive Compatibility for Buyers

That is, buy bids equal private valuations

- Consider a single seller
- Consider two buyer agents A_1 and A_2, with private valuations v_1 and v_2, bidding b_1 and b_2
- If $b_1 > b_2$, A_1 wins and pays b_2
  - A_1's utility in that case is $v_1 - b_2$: could be positive or negative
- If $b_1 < b_2$, A_1 loses the auction: utility $= 0$ (assuming no bidding costs)
- If $(v_1 - b_2) > 0$ (i.e., $v_1 > b_2$)
  - Then A_1 benefits by maximizing $\text{Prob}(b_1 > b_2)$
    - *Underbid:* likelier to lose, but would pay the same price if it wins
  - Else A_1 benefits by minimizing $\text{Prob}(b_1 > b_2)$
    - *Overbid:* likelier to win, but may pay more than the valuation
- Thus, setting the bid equal to valuation is the best strategy

Vickrey Incentive Compatibility: 2

- If A_1 wins, what A_1 pays depends on bids by other agents
- A_1 should try to
  - Win when it would benefit by winning
  - Lose when it would suffer by winning

How do the above ideas apply when a buyer is bidding for multiple units of the same item?

M^{th} and $(M+1)^{st}$ Price Auctions

- Vickrey = $(M+1)^{st}$ price, with one unit for sale
- For single-unit buyers, $(M+1)^{st}$ price induces truthfulness
- For multiunit buyers, NO!
  - A buyer may artificially lower some bids to lower the price for other bids

Dominant Strategies

One which yields a greater payoff for the agent than any of its other strategies (regardless of what others bid)

- Under Vickrey auctions, the dominant strategy for a *buyer* is bidding according to its true value
- Under first-price auctions, the dominant strategy for a *seller* is to bid its true value

Multiunit Auctions

- Multiunit bids are *divisible* when not necessarily the whole set needs to be bought or sold
- When multiunit bids are divisible,
  - Treat multiunit bids as multiple copies of single-unit bids
  - If indivisible, e.g., sets of two or four tires, then treat as bundled goods

Desirable Properties of Markets

- Efficient: the one values it most gets it
  - If seller's valuation $<$ buyer's valuation, they trade
- Truthful
  - Rational to bid true valuation for both sellers and buyers
- Individually rational
  - No participant is worse off for participating
- Budget balanced, i.e., no subsidy from the market: Σpayment $=$ Σrevenue
  - Seller receives what the buyer pays

Can all of the above be satisfied?

Impossibility Result

Given a sealed buy bid b and a sealed sell bid s (Meyerson & Satterthwaite)

- Valuations of each from overlapping distributions
- Ultimately buyer pays p_b and seller gets p_s
  - For truthfulness, $p_b = s$ and $p_s = b$
  - But the deal happens only if $b > s$, else irrational
  - Thus buyer pays less than the seller receives, i.e., a deficit!

That is, *subsidize* or relax another requirement

McAfee's Dual Price Auction: 1

- Let p be a price in the equilibrium range
  - That is, M^{th} to $(M+1)^{st}$
  - Let's choose the midpoint to be specific
- Omits the lowest buyer at or above M^{th} and the highest seller at or below $(M+1)^{st}$

Which of the above properties does the dual price auction violates?

McAfee's Dual Price Auction: 2

- Individually rational
- Promotes truthfulness
- Budget balanced
- *Inefficient*
  - Discards the lowest valued match
  - Not good if it is the only one

Continuous Double Auctions

As in stock markets and prediction markets

- Multiple sellers and buyers, potentially with multiple sell and buy bids each
- *Bid* quote: what a seller needs to offer to form a match
- *Ask* quote: what a buyer needs to offer to form a match
- *Clears* continually:
  - The moment a buyer and seller agree on a price, the deal is done and the matching bids are taken out of the market
  - Possibly, a moment later a better price may come along, but it will too late then
- The *bid-ask spread* represents the difference between the buyers and the sellers

Prediction Markets: 1

Combining markets with crowdsourcing

- A market computes an equilibrium price for a commodity
- Suppose the commodity were a prediction
  - A *security*: abstract like a share
- If we could trade on the predictions, the equilibrium would correspond to the median
  - Equilibrium because supply equals demand at the median
  - Half the bids are above: those bidders would buy at the median
  - Half the bids are below: those bidders would sell at the median

Prediction Markets: 2

Continual: absorbs dynamic information
Galton's case was a one-shot sealed bid

- Predictions as commodities could be traded
- Payoff when the prediction settles: becomes reality
- Whoever has better knowledge than the market
  - Sells if their estimate is that it is less likely
  - Buys if their estimate is that it is less likely
  - They profit from their knowledge
  - The market's price shifts accordingly
- Those who mistakenly think they are knowledgeable
  - Lose money
  - Assumed to cancel out
- The market price at any time is the best estimate

Prediction Markets for Probabilities

Winner Takes All

- A prediction is for a future event
- Whoever has better knowledge than the market
  - Price of oil on December 2
  - Rainfall will exceed 1 inch at RDU on October 31
  - Who will win the Oscar for Best Motion Picture in 2021
- If so, owner of the security cashes out for $1
- If not, owner of the security gets nothing
- Computes probability of the event

Prediction Markets: Main Types

Winner takes all	Pays $1 if and only if the (Boolean) event occurs	Reveals market expectation of the probability
Index	Pays $1 for each percentage point of the event Real-valued event, normalized to [0, 1], such as fraction of votes received by a candidate	Reveals market expectation of the mean value of the event
Spread	Sold for $1; Pays $2 if the event value beats the spread, else zero	Reveals market expectation of the median value of the event

Exercise: Limitations of Prediction Markets

Identify the key assumptions and when those assumptions may be violated

- A prediction is for a future event
- Whoever has better knowledge than the market
  - Price of oil on December 2
  - Rainfall will exceed 1 inch at RDU on October 31
  - Who will win the Oscar for Best Motion Picture in 2021
- If so, owner of the security cashes out for $1
- If not, owner of the security gets nothing
- Computes probability of the event

Limitations of Prediction Markets

- Central tendency
- Potential irrational behavior by participants
- Manipulation of participants
- Side bets
  - Sethi and Rothschild's study of the 2012 US Presidential Elections
  - The Romney security stayed higher than the polls

Auction Management: Bidding

Bidding rules to govern, e.g.,

- Whose turn it is
- What the minimum acceptable bid is, e.g., increments
- What the reserve price is, if any

Compare these for outcry, silent, sealed bid, and continuous auctions

Auction Management: Information

What information is revealed to participants?

- Bid value (not in sealed bid auctions)
- Bidder identity (not in sealed bid auctions or stock exchanges)
- Winning bid or current high bid
- Winner
- How often, e.g., once per auction, once per hour, any time, and so on

Auction Management: Clearing

Bids are cleared when they are executed and taken out of the market

- What defines a deal: how are bids matched?
    - What prices? If uniform, then matching is not relevant
    - Who?
- How often?
- Until when?