



Estimating positive surveys from negative surveys

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ABSTRACT

The negative survey is an emergent survey method, which could protect sensitive data and individual privacy. Because positive survey results are needed in most situations, it is essential to estimate positive surveys from negative surveys. However, the traditional method for reconstructing positive surveys from negative surveys could return negative values (i.e. less than zero), and obviously this is impractical. In this paper, two novel methods to estimate positive surveys from negative surveys are proposed. Both methods can return nonnegative and proper values, and their results are identical. Simulation experiments demonstrate that the proposed two methods return more reasonable results than the traditional method.

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1. Introduction

Inspired by the self–nonself discrimination mechanism in immunology (Janeway et al., 1999), Esponda et al. proposed the negative representation of information (Esponda et al., 2004a,b). The negative representation of information has some unique properties, and it can be used in data security (Esponda, 2008). Meanwhile, Esponda introduced the negative representation of information into data collection, and proposed *Negative Surveys*, which could protect sensitive data and individual privacy (Esponda, 2006).

In a negative survey, participants are requested to select a category that does NOT agree with the fact (Esponda, 2006; Esponda and Guerrero, 2009). In a traditional survey, the participants are requested to select a category that agrees with the fact; such surveys are also called *Positive Surveys*. For convenience, a category that agrees with the fact is defined as a *positive category*, while a category that does NOT agree with the fact is defined as a *negative category* (Esponda, 2006).

Therefore, negative surveys, which only collect part of the negative categories, can protect individual privacy. Following the work by Esponda (2006), some other work related to negative surveys has been done. Xie et al. (2011) proposed the *Gaussian Negative Survey* (GNS). Unlike a negative survey, where each negative category has an equal chance of being selected (Esponda, 2006), in Gaussian negative surveys, the probabilities of selecting negative categories follow a Gaussian distribution centered at the positive category. Both negative surveys and GNSs can collect data while protecting individual privacy. Compared with negative surveys, GNSs achieve a higher accuracy with lower ability of privacy protection. Based on the negative survey, Horey et al. (2007) implemented anonymous data collection on sensor network platforms.

Generally, in order to effectively make use of the negative survey results, positive surveys are reconstructed from negative surveys. In the work by Esponda (2006), a method to estimate positive surveys from negative surveys has been proposed. However, when this traditional method (Esponda, 2006) is used to estimate positive surveys from negative surveys, it could return negative values (i.e. less than zero), and this is impractical. In fact, in Horey et al. (2007), when the positive

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survey (i.e. the histogram of the original sensor data) is reconstructed from the negative survey with the traditional method, sometimes negative values could appear (see Fig. 1 in Horey et al. (2007) for details).

In this paper, two novel methods are proposed to estimate positive surveys from negative surveys: both methods return nonnegative and more reasonable results. Simulation experiments are done to demonstrate the performance of the proposed methods.

The rest of this paper is organized as follows. Section 2 describes the traditional method for estimating positive surveys from negative surveys and its limitations in detail. In Section 3, two novel ways are proposed to estimate positive surveys from negative surveys. Simulation results are shown in Section 4. Section 5 includes some discussions. Finally, the work of this paper is briefly concluded in Section 6.

2. Background

The negative survey was first proposed by Esponda (2006). For convenience, in this section, the background of the negative survey is introduced (Esponda, 2006; Esponda and Guerrero, 2009; Horey et al., 2007; Xie et al., 2011).

Assume that the total number of participants in the survey is N , the number of categories is c ($c > 1$), and the result obtained from the negative survey is $R = (r_1, r_2, r_3, \dots, r_c)$. Here, r_i represents the total number of participants who select category i in the negative survey. The corresponding positive survey result is $T = (t_1, t_2, t_3, \dots, t_c)$, and t_i represents the total number of participants who select category i in the positive survey. Obviously, R and T should satisfy the following constraints (Esponda, 2006; Esponda and Guerrero, 2009).

$$\sum_{i=1}^c r_i = N \quad (1)$$

$$r_i \geq 0, \quad i = 1, 2, \dots, c$$

$$\sum_{i=1}^c t_i = N \quad (2)$$

$$t_i \geq 0, \quad i = 1, 2, \dots, c.$$

Let q_{ij} be the probability of participants belonging to positive category i but selecting category j in the negative survey; Q represents the corresponding probability matrix. Thus,

$$Q = \begin{bmatrix} 0 & q_{12} & \cdots & q_{1c} \\ q_{21} & 0 & \cdots & q_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ q_{c1} & q_{c2} & \cdots & 0 \end{bmatrix}$$

$$\sum_{j=1, j \neq i}^c q_{ij} = 1, \quad i = 1, 2, \dots, c.$$

For each category i , all participants positively belonging to category i select category j with probability q_{ij} . Therefore, the number of participants selecting category j can be calculated as follows (Esponda, 2006; Esponda and Guerrero, 2009).

$$r_j = \sum_{i=1}^c t_i q_{ij}. \quad (3)$$

That is

$$R = TQ. \quad (4)$$

Thus

$$T = RQ^{-1}. \quad (5)$$

From formula (5), therefore, the positive survey could be calculated from the negative survey. For convenience, this method is named *NStoPS* in this paper.

In the general negative survey, also named the *Uniform Negative Survey* (UNS) in Xie et al. (2011), where each negative category has the same probability of being selected, q_{ij} could be calculated using the following formula (Esponda, 2006; Esponda and Guerrero, 2009).

$$q_{ij} = \begin{cases} \frac{1}{c-1}, & i \neq j \\ 0, & i = j. \end{cases} \quad (6)$$

According to formulas (2), (3) and (6),

$$t_j = N - (c-1)r_j. \quad (7)$$

From formula (7), it can be observed that t_i is less than zero when r_i is greater than $N/(c - 1)$. Therefore, this traditional method is not practical.

As for the GNS (Xie et al., 2011), let $f(j; i, \sigma^2)$ be the continuous probability density function for a Gaussian distribution, which is centered at a positive category i with a standard deviation σ . Then q_{ij} could be calculated using the following formula (Xie et al., 2011).

$$q_{ij} = \begin{cases} \frac{f(j; i, \sigma^2)}{\sum_{k=1, k \neq i}^c f(k; i, \sigma^2)}, & i \neq j \\ 0, & i = j. \end{cases} \tag{8}$$

Although this probability matrix Q is different from that in formula (6), the corresponding positive survey results can still be calculated using formula (5). Therefore, it is also possible for GNSs to generate negative values when the *NStoPS* method is used to reconstruct the positive survey. This will be discussed in Section 5 in this paper.

3. From negative surveys to positive surveys

In this section, two novel methods are proposed to estimate positive surveys from negative surveys. Both methods return nonnegative and proper results. Both methods are iterative, and the methods presented in Sections 3.1 and 3.2 are named *NStoPS-I* and *NStoPS-II*, respectively. In Section 3.3, the proposed two methods and the traditional method are compared.

For convenience, in this section, it is supposed that each negative category has the same probability of being selected. As for the GNSs, where the negative category is selected with a Gaussian distribution, the corresponding estimation methods will be discussed in Section 5.

3.1. The proposed *NStoPS-I* method

Let m_{ij} be the probability of selecting category i in the negative survey but belonging to positive category j ; M represents the probability matrix. Thus

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1c} \\ m_{21} & m_{22} & \cdots & m_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ m_{c1} & m_{c2} & \cdots & m_{cc} \end{bmatrix}.$$

According to the definition of the negative survey, m_{ij} should satisfy the following equations.

$$\begin{cases} m_{ii} = 0, & i = 1, 2, \dots, c, \\ \sum_{i=1}^c m_{ij} = 1, & j = 1, 2, \dots, c. \end{cases}$$

The participants that select category i in the negative survey positively belong to other categories except category i . If each negative category has the same probability of being selected, m_{ij} is proportional to the number of participants who belong to positive category j (i.e. t_j), and it can be calculated with the following formula.

$$m_{ij} = \begin{cases} \frac{t_j}{N - t_j}, & i \neq j \\ 0, & i = j. \end{cases} \tag{9}$$

Let X_{ij} be the number of participants selecting category i in the negative survey, but belonging to positive category j . Therefore, the expectation value of X_{ij} is

$$E(X_{ij}) = r_i m_{ij}. \tag{10}$$

In fact, X_{ij} follows a binomial distribution, i.e. $X_{ij} \sim B(r_i, m_{ij})$. The number of participants who belong to positive category j , i.e. t_j , can be calculated as follows.

$$t_j = X_{1j} + X_{2j} + \cdots + X_{cj} = \sum_{i=1}^c X_{ij}.$$

So the estimated value of t_j is

$$\hat{t}_j = \sum_{i=1}^c E(X_{ij}) = \sum_{i=1}^c r_i m_{ij}. \tag{11}$$

Input: The negative survey result $R = (r_1, r_2, r_3, \dots, r_c)$.
Output: The estimated positive survey result $T = (t_1, t_2, t_3, \dots, t_c)$.

```

1. for i ← 1 to c
2.    $t_i^{(0)} \leftarrow N/c$ ;
3. n ← 0,  $\varepsilon \leftarrow 1e-4$ ;
4. delta_max ←  $-\infty$ ;
5. for i ← 1 to c
6.   for j ← 1 to c
7.     if  $i \neq j$  then  $m_{ij}^{(n)} \leftarrow \frac{t_j^{(n)}}{N - t_i^{(n)}}$ 
8.     else  $m_{ij}^{(n)} \leftarrow 0$ 
9.   for j ← 1 to c
10.     $t_j^{(n+1)} \leftarrow \sum_{i=1, i \neq j}^c r_i m_{ij}^{(n)}$ ;
11.  $t_j \leftarrow t_j + N - \sum t_j$ ;
12. for j ← 1 to c
13.   if delta_max <  $|t_j^{(n+1)} - t_j^{(n)}|$ 
14.     delta_max ←  $|t_j^{(n+1)} - t_j^{(n)}|$ ;
15. if dt_max <  $\varepsilon$ 
16.   Terminate and output the values of  $T = (t_1, t_2, t_3, \dots, t_c)$ .
17. n ← n + 1;
18. Goto 4;
```

Fig. 1. The *NStoPS-I* algorithm.

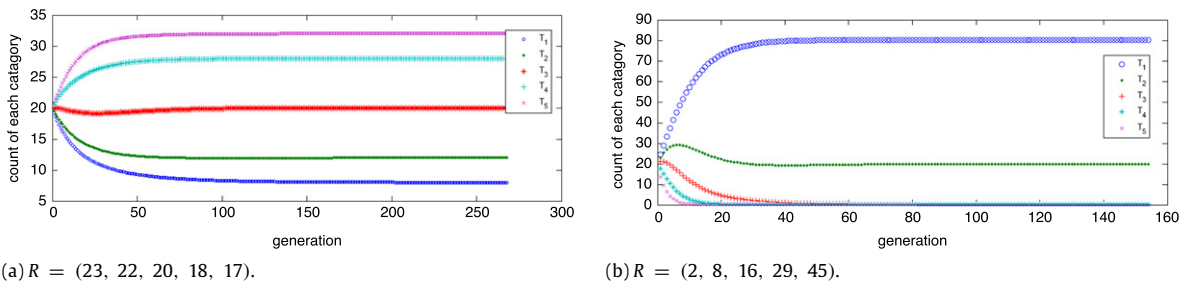


Fig. 2. The variation curves of the positive survey results during iterations.

Although the values of t_j and m_{ij} are unknown, they satisfy formulas (9) and (11), and t_j should also satisfy formula (2). Thus the values t_j and m_{ij} can be calculated through an iterative process. Assume that the number of participants who belong to each positive category, i.e. t_i ($1 \leq i \leq c$), are equal when the iteration begins, namely

$$t_i^{(0)} = \frac{N}{c}.$$

Then formulas (9) and (11) are used repeatedly to update the values of t_j and m_{ij} , until the convergence criterion is met. When the values of t_j ($1 \leq j \leq c$) do not vary with the iteration, the algorithm terminates.

If each negative category has the same probability of being selected, the pseudo code of the algorithm is shown in Fig. 1. For convenience, this algorithm is named *NStoPS-I*.

In Fig. 1, Step 11 is introduced to eliminate the error caused by the float calculation. As for the terminal condition, t_j is regarded as unvarying if the difference between the values of each t_j ($1 \leq j \leq c$) in two adjacent iterations is less than a small constant ε . In this paper, ε is set to 10^{-4} .

Two examples are given in Fig. 2. When the result obtained from the negative survey is $R = (23, 22, 20, 18, 17)$, the variation curves of t_j ($1 \leq j \leq 5$) during iterations are shown in Fig. 2(a). When $R = (2, 8, 16, 29, 45)$, the variation curves of t_j ($1 \leq j \leq 5$) during the iteration are shown in Fig. 2(b). From Fig. 2, it can be seen that the *NStoPS-I* algorithm converges quickly.

3.2. The proposed *NStoPS-II* method

The *NStoPS* method may obtain negative survey results, which do not meet actual situations. The proposed *NStoPS-II* method is based on the idea of adjusting the results of *NStoPS*. The primary idea of the *NStoPS-II* method is given as follows.

First, the positive survey results are calculated using formula (7). Second, in order to get a more accurate result, the negative values in the positive survey results should be fixed to 0. That is to say, no one belongs to those categories. The

Input: The negative survey result $R = (r_1, r_2, r_3, \dots, r_c)$.
Output: The estimated positive survey result $T = (t_1, t_2, t_3, \dots, t_c)$.

```

1. for i ← 1 to c // calculate the positive survey results using formula (7)
2.    $t_i \leftarrow N - (c - 1) * r_i$ ;
3.    $b_i \leftarrow 1$ ;
4.    $N' \leftarrow N$ ;
5.    $c' \leftarrow c$ ;
6.   while (1)
7.     if ( $t_i \geq 0$ , for i ← 1 to c) then break;
8.     for i ← 1 to c
9.       // exclude the categories whose positive survey results are negative
10.      if  $t_i < 0$  then
11.         $t_i \leftarrow 0$ ;
12.         $b_i \leftarrow 0$ ;
13.         $N' \leftarrow N' - r_i$ ;
14.         $c' \leftarrow c' - 1$ ;
15.      for i ← 1 to c // recalculate the positive survey results
16.        if  $b_i = 1$  then
17.           $t_i \leftarrow N' - (c' - 1) * r_i$ ;
18.      for i ← 1 to c // scale the positive survey results
19.        if  $b_i = 1$  then
20.           $t_i \leftarrow t_i / N' * N$ ;
21. Terminate and output the value of  $T = (t_1, t_2, t_3, \dots, t_c)$ .
```

Fig. 3. The *NStoPS-II* algorithm.

negative survey results give no meaningful information for these categories, and should be excluded when calculating the positive survey results. Therefore, third, as for other categories whose results are no less than 0, after the categories whose positive survey results are less than zero are excluded, their positive survey results are calculated with formula (7) again. Fourth, in order to keep the sum of the positive survey unchanged, the computing results obtained by formula (7) need to be scaled. That is to say, the sum of the positive survey is equal to that of the negative survey. If there still exist minus values in the positive survey results, the above procedure is conducted again.

The pseudo code of the algorithm is shown in Fig. 3.

Because the number of categories whose positive survey results are no less than zero always decreases, the *NStoPS-II* algorithm could terminate in finite steps. In particular, if there are only two categories whose positive survey results are no less than zero, the positive survey results calculated according to formula (7) will be nonnegative.

3.3. Comparison with the traditional method

When r_i ($1 \leq i \leq c$) is greater than $N/(c - 1)$, as for the traditional method, the estimated value of t_i is less than zero. As for *NStoPS-I*, since the initial values of t_i are positive, according to formulas (11) and (9), the values of t_j and m_{ij} are nonnegative during the iteration process. Thus, *NStoPS-I* will always return nonnegative results. As for the *NStoPS-II*, the algorithm will terminate when all the values in the positive survey results are nonnegative. Therefore, *NStoPS-II* will also always return nonnegative results.

When all r_i are less than $N/(c - 1)$, both the proposed methods and the traditional method return the same results. As for *NStoPS-II*, the algorithm will terminate without iteration, so the results are the same as with the traditional method. As for *NStoPS-I*, it can be seen that formula (7) satisfies equations consisting of formulas (1), (2), (9) and (11). Thus, when the values of t_i reach the value in formula (7), *NStoPS-I* terminates.

Simulation results in Section 4 also verify the above analysis.

4. Simulation

In order to compare *NStoPS* (Esponda, 2006) with the methods proposed in this paper, simulation experiments are conducted in this section. The experimental settings are described as follows.

First, the positive survey results are generated. In Horey et al. (2007), the positive survey results are sampled with three different distributions, i.e. a uniform distribution, a normal distribution, and an exponential distribution. In this paper, we suppose that the positive survey results are sampled with four different distributions, i.e. a uniform distribution, a normal distribution, an exponential distribution, and a log-normal distribution. For each distribution, N random numbers are generated which obey the corresponding distribution to represent the choices of N participants.

Second, a negative survey is conducted based on the negative survey rules. Namely, each participant selects a negative category randomly. Then the negative survey result is obtained.

Finally, the estimated positive survey results are calculated with *NStoPS*, *NStoPS-I*, and *NStoPS-II*, respectively.

The parameters for the four distributions are shown in Table 1. When the total number of participants $N = 500$, and the number of categories $c = 5$, both the original positive survey results and the negative survey results are given in Table 2. For convenience, the original positive survey results and the estimated values under the four different distributions are shown in Fig. 4(a)–(d).

Table 1
Parameters for the four distributions.

Distribution	Parameters
Uniform	$U(1, 5)$
Normal	$N(3, 0.6667)$
Exponential	$e(0.6886)$
Log-normal	$\text{Log-}N(0.8959, 0.2310)$

Table 2
The original positive survey results and negative survey results.

Distribution	Original positive survey results	Negative survey results
Uniform	(84, 102, 90, 112, 112)	(107, 82, 101, 104, 106)
Normal	(24, 229, 217, 30, 0)	(120, 55, 64, 124, 137)
Exponential	(380, 101, 17, 2, 0)	(31, 95, 121, 122, 131)
Log-normal	(97, 300, 95, 8, 0)	(96, 50, 105, 118, 131)

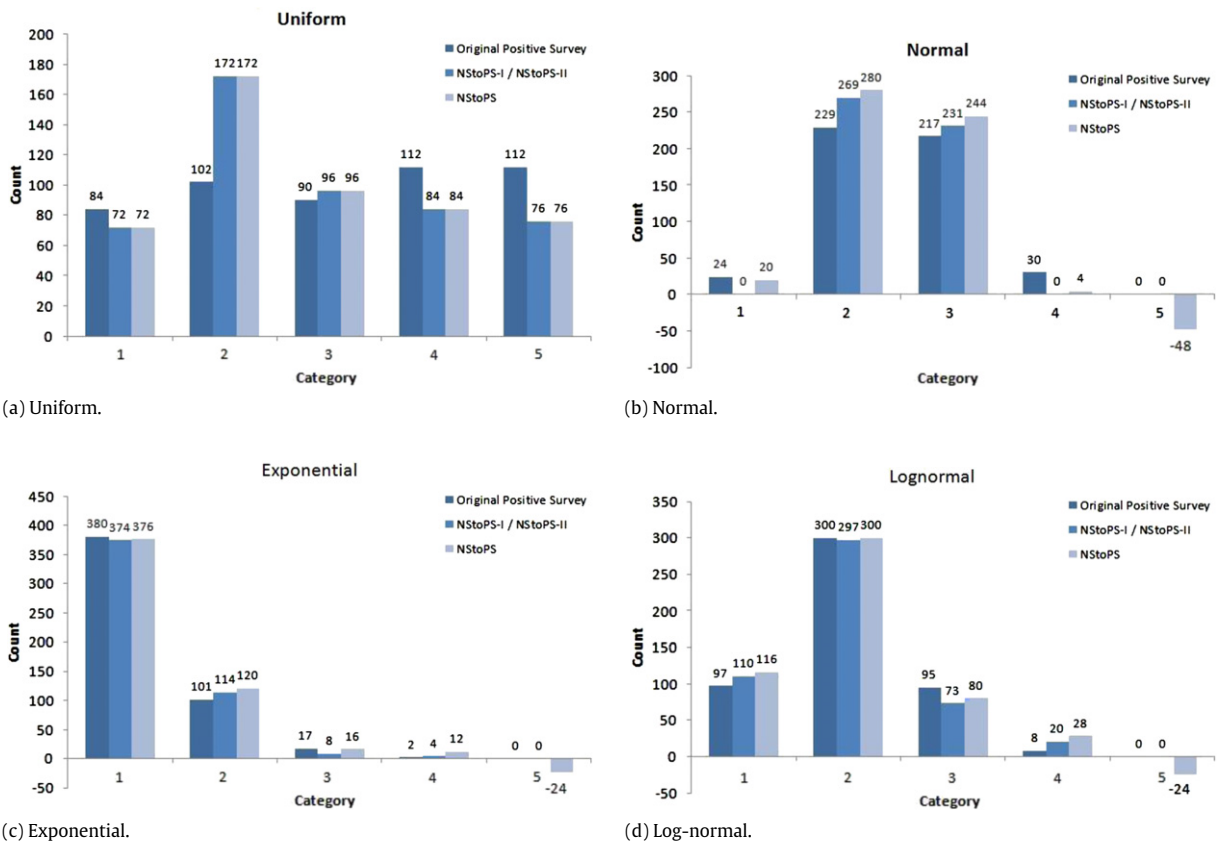


Fig. 4. The estimated positive survey results under different distributions.

From Fig. 4, when *NStoPS* returns nonnegative values, it can be seen that *NStoPS-I*, *NStoPS-II*, and *NStoPS* return the same results.

Also, it can be observed that the proposed *NStoPS-I* and *NStoPS-II* could return nonnegative results when *NStoPS* returns negative values. When *NStoPS* returns negative values, the results returned by both methods proposed in this paper are much closer to the original positive survey results. Therefore, the proposed *NStoPS-I* and *NStoPS-II* are applicable in practical situations.

5. Discussion

With the development of the Internet, both data security and privacy protection become more and more important. The negative representation of information (Esponda et al., 2007, 2004b) is a new and promising method for data security and

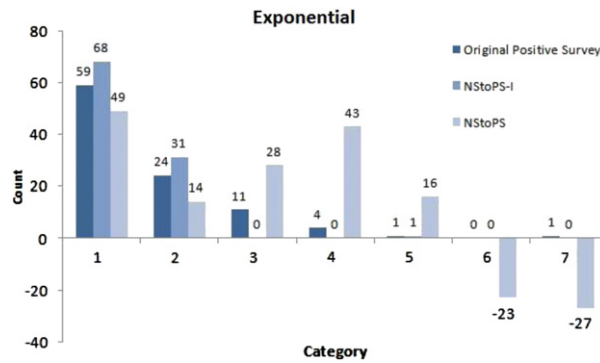


Fig. 5. Results of the NStoPS and NStoPS-I for a GNS.

privacy protection. The negative survey is based on the negative representation of information. This method could protect the privacy of participants effectively while collecting information.

A major problem in the negative survey is how to estimate positive surveys from negative surveys. The method in Esponda (2006) returns negative values and has a relatively low accuracy in some occasions. In this paper, two novel methods are proposed to estimate positive surveys from negative surveys, which return proper results in all occasions.

In Sections 3 and 4, the proposed methods are given and tested when each negative category has the same probability of being selected. However, NStoPS-I is also applicable to other situations. Here, the GNS is taken as an example. Although the GNS has a higher accuracy, it could still return negative values (see Fig. 5).

As for NStoPS-I, if the survey is done by the GNS (Xie et al., 2011), Step 7 in Fig. 1 should adopt formula (12) instead of (9). In other words, as for GNSs (Xie et al., 2011), m_{ij} can be calculated with formula (12). And q_{ij} is calculated with formula (8) in Section 2.

$$m_{ij} = \frac{q_{ji}t_j}{\sum_{k=1}^c q_{ki}t_k} \tag{12}$$

As for GNS, the comparison between NStoPS and NStoPS-I is shown in Fig. 5. It can be seen from Fig. 5 that NStoPS-I returns better results than NStoPS. In this experiment, the total number of participants $N = 100$, the number of categories $c = 7$, and an exponential distribution $e (1.0329)$ is adopted to sample the positive survey.

It should be noted that the proposed NStoPS-II is also not directly applicable to GNSs. As for GNSs, its probability matrix Q is different from that for the general negative survey. In particular, even if no one belongs to a positive category, the negative survey results of such a category have different impacts on the estimation of other categories. For example, if no one belongs to positive category 3 but a lot of participants select category 3 in GNSs, this means that many people positively belong to category 2 or category 4. Therefore, NStoPS-II should be modified according the probability matrix Q of GNSs. This needs further study.

6. Conclusion

In this paper, how to estimate a positive survey from a negative survey is analyzed. Two novel methods, i.e. NStoPS-I and NStoPS-II, are proposed. Compared to the traditional NStoPS, both NStoPS-I and NStoPS-II can return nonnegative values, which are much closer to practical situations. Simulation experiments were conducted to illustrate the difference between these methods. The proposed NStoPS-I is applicable to all kinds of negative survey, including Uniform Negative Surveys and Gaussian Negative Surveys.

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