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SOME SUPER-CLASSICS OF MATHEMATICS

Joseph A. Schatz
Sandia Corporation
Albuquerque, New Mexico

This collection of thirty-seven papers was chosen from the material which was collected by the author in the course of compiling the *Mathematics Citation Index* [3]. A citation index is a list of papers with each item on the list followed by a list of the papers which have cited (referred to) the given item. The *Mathematics Citation Index* was compiled from the references in approximately twenty-five thousand papers (roughly two-hundred thousand citations) taken from forty-eight serials published during the period 1950-1965. This is about five percent of the total mathematical literature, about ten percent of the mathematical literature published since *Mathematical Reviews* started, and about twenty percent of the periodical literature for the period covered. By definition, the super-classics are the papers which were cited fifty or more times in this corpus. The following thirty-seven super-classics were cited 2,549 times for an average of 68.9 citations per paper.

1. Lars Ahlfors and Arne Beurling, Conformal invariants and function-theoretic null-sets, Acta Math. 83 (1950), 101-129. MR 12, #171.

2. Armand Borel, Sur la cohomologie des espaces fibrés principaux et des espaces homogènes de groupes de Lie compacts, Ann. of Math. (2) 57 (1953), 115-207. MR 14, #490.

3. E. Čech, On bicompact spaces, Ann. of Math. (2) 38 (1937), 823-844.

4. I. S. Cohen, On the structure and ideal theory of complete local rings, Trans. Amer. Math. Soc. 59 (1946) 54-106. MR 7, #509.

5. Jean Dieudonné et Laurent Schwartz, La dualité dans les espaces ($\mathcal{F}$) et ($\mathcal{LF}$), Ann. Inst. Fourier (Grenoble) 1 (1949), 61-101 (1950). MR 12, #417.

6. Samuel Eilenberg and Saunders Mac Lane, Cohomology theory in abstract groups. I, Ann. of Math. (2) 48 (1947), 51-78. MR 8, #367.

7. Lars Gårding, Dirichlet's problem for linear elliptic partial differential equations, Math. Scand. 1 (1953), 55-72. MR 16, #366.

8. I. M. Gel'fand, Normierte Ringe, Mat. Sb. 9 (51) (1941), 3-24. MR 3, #51.

9. K. Gödel, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme. I, Monatsh. Math. Phys. 38 (1931), 173-198.

10. Alexandre Grothendieck, Sur quelques points d'algèbre homologique, Tôhoku Math. J. (2) 9 (1957), 119-221. MR 21 #1328.

11. Marshall Hall, Projective planes, Trans. Amer. Math. Soc. 54 (1943), 229-277. MR 5, #72.

12. P. Hall, A contribution to the theory of groups of prime-power order, Proc. London Math. Soc. (2) 36 (1933), 29-95.

13. Edwin Hewitt, Rings of real-valued continuous functions. I, Trans. Amer. Math. Soc. 64 (1948), 45-99. MR 10, #126.

14. Lars Hörmander, On the theory of general partial differential operators, Acta Math. 94 (1955), 161-248. MR 17, #853.

15. Kenkichi Iwasawa, On some types of topological groups, Ann. of Math. (2) 50 (1949), 507-558. MR 10, #679.

16. N. Jacobson, The radical and semi-simplicity for arbitrary rings, Amer. J. Math. 67 (1945), 300-320. MR 7, #2.

17. Shizuo Kakutani, Concrete representation of abstract (M)-spaces. (A characterization of the space of continuous functions.), Ann. of Math. (2) 42 (1941), 994-1024. MR 3, #205.

18. M. G. Kreĭn and M. A. Rutman, Linear operators leaving invariant a cone in a Banach space, Uspehi Mat. Nauk 3 (1948), no. 1 (23), 3-95; English transl., Amer. Math. Soc. Transl. no. 26 (1950); Reprint of translation, Amer. Math. Soc. Transl. (1) 10 (1962), 199-325. MR 10, #256; MR 12, #341.

19. J. Leray et J. Schauder, Topologie et équations fonctionnelles, Ann. Sci. École Norm. Sup. (3) 51 (1934), 45-78.

20. Edwin E. Moise, Affine structures in 3-manifolds. V. The triangulation theorem and Hauptvermutung, Ann. of Math. (2) 56 (1952), 96-114. MR 14, #72.

21. F. J. Murray and J. von Neumann, On rings of operators, Ann. of Math. (2) 37 (1936), 116-229.

22. Louis Nirenberg, Remarks on strongly elliptic partial differential equations, Comm. Pure Appl. Math. 8 (1955), 649-675. MR 17, #742.

23. D. Rees, On semi-groups, Proc. Cambridge Philos. Soc. 36 (1940), 387-400. MR 2, #127.

24. Jean-Pierre Serre, Homologie singulière des espaces fibrés. Applications, Ann. of Math. (2) 54 (1951), 425-505. MR 13, #574.

25. Jean-Pierre Serre, Groupes d'homotopie et classes de groupes abéliens, Ann. of Math. (2) 58 (1953), 258-294. MR 15, #548.

26. Jean-Pierre Serre, Faisceaux algébriques cohérents, Ann. of Math. (2) 61 (1955), 197-278. MR 16, #953.

27. A. H. Stone, Paracompactness and product spaces, Bull. Amer. Math. Soc. 54 (1948), 977-982. MR 10, #204.

28. M. H. Stone, The theory of representations for Boolean algebras, Trans. Amer. Math. Soc. 40 (1936), 37-111.

29. M. H. Stone, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41 (1937), 375-481.

30. René Thom, Espaces fibrés en sphères et carrés de Steenrod, Ann. Sci. École Norm. Sup. (3) 69 (1952), 109-182. MR 14, #1004.

31. René Thom, Quelques propriétés globales des variétés différentiables, Comment. Math. Helv. 28 (1954),17-86. MR 15, #890.

32. A. D. Wallace, The structure of topological semigroups, Bull. Amer. Math. Soc. 61 (1955), 95-112. MR 16, #796.

33. H. Weyl, Über gewöhnliche Differentialgleichungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen, Math. Ann. 68 (1910), 220-269.

34. George W. Whitehead, A generalization of the Hopf invariant, Ann. of Math. (2) 51 (1950), 192-237. MR 12, #847.

35. J. H. C. Whitehead, Simplicial speces, nuclei and m-groups, Proc. London Math. Soc. (2) 45 (1939), 243-327.

36. J. H. C. Whitehead, Combinatorial homotopy. I, Bull. Amer. Math. Soc. 55 (1949), 213-245. MR 11, #48.

37. E. Witt, Treue Darstellung Liescher Ringe, J. Reine Angew. Math. 177 (1937), 152-160.

One of the most striking features of the list of super-classics is the heavy representation of algebraic topology. Since *Mathematical Reviews* had no heading for algebraic topology until volume 17 (1956), and the last paper I would classify as algebraic topology was reviewed in volume 16 (1955), none of these were classified as algebraic topology in *Mathematical Reviews*. The next largest representation of papers is in functional analysis, followed closely by algebra. The list of journals represented is even more strongly skewed than the list of subjects. The *Annals of Mathematics* accounts for eleven of the thirty-seven papers, and is represented by super-classics in functional analysis and algebra as well as those in algebraic topology. It is interesting to note that the *Annals*' popularity in citations is not a new fact. C. H. Brown [1] reports that in the period from 1929 to 1954 the *Annals of Mathematics* went from the tenth to the first most cited mathematical journal, and that the *Transactions of the American Mathematical Society* was the second most cited mathematical journal. Of the super-classics herein discussed, eleven appeared in the *Annals*, five in the *Transactions*, three in the *Bulletin of the American Mathematical Society*, two each in *Acta Mathematica* and *Annales Scientifiques de l'École Normale Supérieure*, and one each in fourteen other journals.

There were no papers in the super-classics with more than two authors, although six were papers by two authors. This is remarkably typical of the entire corpus.

Until the end of 1965, *Mathematical Reviews* had listed about 171,000 items. Kenneth O. May [2] has estimated the number of items published before *Mathematical Reviews* started as 235,000. Thus, 24.7 percent of the journal-like citations in the *Mathematics Citation Index* were to the earlier 58.9 percent of the literature. For the super-classics, the corresponding figures are 664 citations to the ten papers published before *Mathematical Reviews* began (or 27 percent), and 1,885 citations to papers published afterwards. Of the citations of the super-classics 26.1 percent were citations of papers published before *Mathematical Reviews* started. These figures suggest that although mathematics has a substantial research front, it also has a healthy regard for at least some of the older periodical literature.

Citation counts furnish information on the nature of scientific literature which can be obtained in no other way. It would be useful to know, for instance, what percentage of the literature twenty or more years old will ever be used again. Citation counts give us a method of attacking questions like this. Unless care is exercised, however, the information obtained can be considerably distorted. In particular, failure to "clump" different descriptions of the same paper results in overestimates of the number of papers cited and underestimates for the average number of citations per paper. For example, there were 318 different descriptions of the thirty-seven super-classics; if these were thought to be 318 different papers, the conclusion would be that they were cited an average of 8.01 times. This is one of the major reasons why the small amount of information available on the use made of scientific literature should be viewed with considerable skepticism. The reasons why a given paper is described in many ways are the different ways of referring to a given journal, variations in an author's initials or name, difficulty of deciding which year a given volume of a journal was issued, missing information, and incorrect information. The question is not whether these things happen, but how frequently.

Twenty-seven of the super-classics were reviewed by *Mathematical Reviews*. Most of these reviews give the reader no indication of the "exalted" nature of these papers. It is interesting to speculate whether this is more a result of the unpredictability of the future, or of the intentional blandness of the reviews. If the latter is the case, perhaps this blandness has been overdone. This comment is offered with considerable trepidation, since I feel that one should exercise extreme caution in changing a product as excellent as *Mathematical Reviews*.

The author is aware that this report on the super-classics, while based on a large number of citations, is biased by the selection of journals used in compiling the information. They were the journals readily available to the author, and the results might have been quite different if a different selection had been made.

References

[1] C. H. Brown. *Scientific Serials*, *ACRL Monograph* Number 16. Association of College and Reference Libraries, Chicago, 1956.

[2] Kenneth O. May. Quantitative growth of the mathematical literature. *Science*, 154:1672–1673, 1966. **MR** 40, #7070.

[3] Joseph A. Schatz. A mathematics citation index. Research Report SC-RR-70-910, Sandia Laboratories, 1970.